OA3602

*Search Theory and Detection*

Discrete Detection Models

**Objectives:**
- Glimpse probabilities
- Escape probability, revisited
Defining detection rate

The modeling of detections bridges coverage and search
Glimpse Probability

• General assumptions:
  – Glimpses at different times are independent
    • Probability of getting positive detection at one time doesn’t affect probability of getting positive detection at next (non-overlapping) time
  – Glimpses at different locations are independent
    • Sensor doesn’t “change” characteristics depending on where it looks
Discrete Search

\[
f(n) = \begin{cases} 
\gamma(1), & \text{if } n = 1 \\
\gamma(n) \prod_{k=1}^{n-1} (1 - \gamma(k)), & \text{if } n \geq 2 
\end{cases}
\]

Detection rate or “glimpse” probability for discrete looks
Cumulative Detection Probability (CDP)

From (discrete) coverage factor to cumulative detection probability

\[ F(n) = 1 - \prod_{k=1}^{n} (1 - \gamma(k)) \]

Coverage factor (for coverage) IS
Cumulative Detection Probability (for search)

Verify (from Lecture 7 notes):

\[ f(n) \overset{?}{=} F(n) - F(n - 1) \]

\[ F(n) \overset{?}{=} \sum_{k=1}^{n} f(k) \]
Special case: Constant detection rate

*Related to the Bernoulli stochastic process*

Suppose \( \gamma(n) = \gamma = \text{constant}, \quad \forall n \)

Then \( f(n) = \)

\( F(n) = \)

This is the **geometric** distribution!

- Governs the result of repeated Bernoulli trials
  (i.e., Bernoulli stochastic process)
  - "First success of heads in string of coin flips"
Recursion expressions

Enable calculations as a function of values at different time steps

Recall: \( f(n) = \gamma(n) \prod_{k=1}^{n-1} (1 - \gamma(k)) \)

\[
f(n + 1) = f(n) \frac{(1 - \gamma(n)) \gamma(n+1)}{\gamma(n)}
\]

Recall: \( F(n) = 1 - \prod_{k=1}^{n} (1 - \gamma(k)) \)

\[
F(n + 1) = 1 - (1 - F(n)) (1 - \gamma(n+1))
\]
Example: Independent glimpses

Consider five independent glimpses with varying (given) detection rates:

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma(n)$</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>$f(n)$</td>
<td></td>
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Recall recursion expressions:

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<td>γ(n)</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>f(n)</td>
<td>0.2</td>
<td>0.32</td>
<td>0.288</td>
<td>0.0960</td>
<td>0.0384</td>
</tr>
<tr>
<td>F(n)</td>
<td>0.2</td>
<td>0.52</td>
<td>0.808</td>
<td>0.904</td>
<td>0.9424</td>
</tr>
</tbody>
</table>

Recall recursion expressions:

\[
f(n + 1) = f(n) \frac{(1 - \gamma(n))\gamma(n + 1)}{\gamma(n)}
\]

\[
F(n + 1) = 1 - (1 - F(n))(1 - \gamma(n + 1))
\]

Recall standard expressions:

\[
f(n) = \gamma(n) \prod_{k=1}^{n-1} (1 - \gamma(k))
\]

\[
F(n) = 1 - \prod_{k=1}^{n} (1 - \gamma(k))
\]
Comments on glimpse probabilities

*Using example*

1. Glimpse probabilities, \( \gamma(n) \), do not form a distribution for \( N \)
   - Recall discrete r.v. \( N \) is # of glimpses till getting a positive detection)
   - Example:

2. Maximum glimpse probability may not be maximum probability of detection
   - Example: Max glimpse probability at \( n=3 \), max detection prob at \( n=2 \)

3. \( f(n) \) IS a probability distribution on \( N \), but not necessarily a *proper* distribution
Recalling escape probabilities

*Im proper distribution*

But a lot of tools rely on $F(\infty) = 1$ (proper distribution)!

- Need to “re-normalize”

> Simply condition on the event that detection is made by the total number of glimpses
Conditional Probability

*Foundation of the Law of Total Probability*

Recall the Law of Total Probability

\[
p(x) = \sum_y p(x|y)p(y)
\]

\[
\Pr \left[ N = n \mid \text{positive detection by } n \to \infty \right] = \frac{f(n)}{F(\infty)}
\]
Normalizing the CDP

Divide by $F(\infty)$

Remember conditional probability implies assumptions

- “With great power comes great responsibility”
- Always be aware of and clearly state the assumptions used
Conditional Expectation

*How does an improper distribution affect the expectation?*

We have the Law of Total Expectation

\[ E_X [X] = E_Y [E_X [X | Y]] = \sum_y p(y) E_X [X | Y] \]

Proof:
Expected Time Till First Detection

Divide by $F(\infty)$

$$
\mathbb{E} \left[ N \mid \text{positive detection} \quad \text{by } n \to \infty \right] = \frac{\mathbb{E} [N]}{F(\infty)}
$$
Parting shots

• Today we learned...
  – Discrete detections ("glimpse probabilities")
  – Discrete cumulative detection probability

• Next time...
  – Continuous detections
  – Continuous cumulative detection probability

Don’t forget to turn in your mud cards!